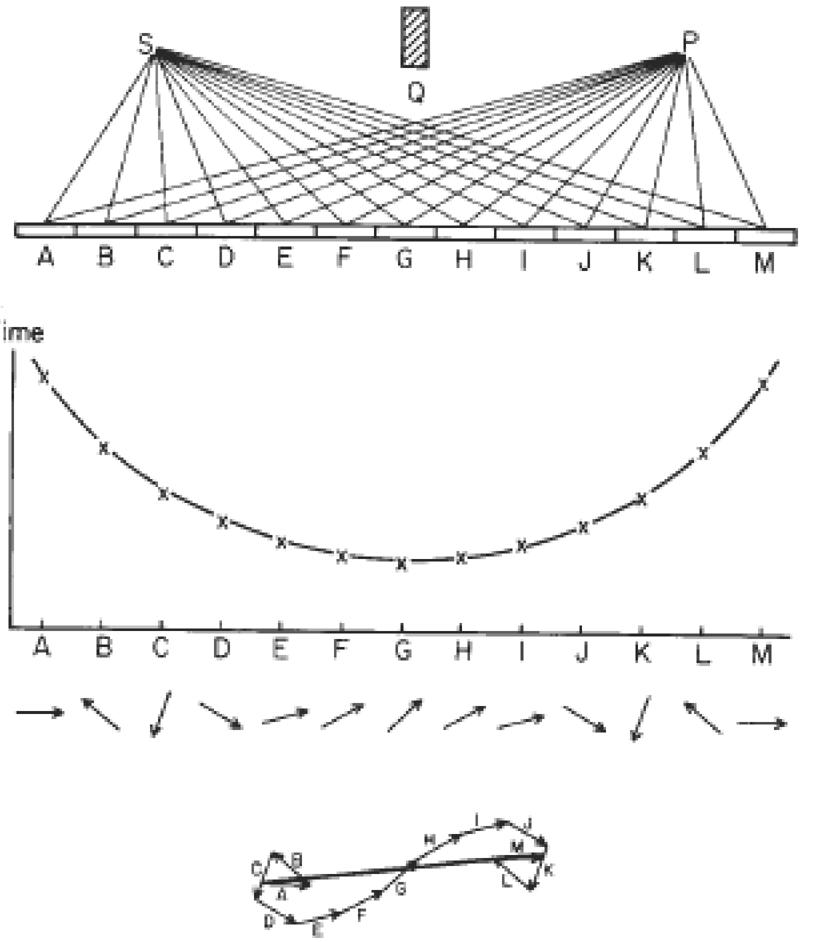
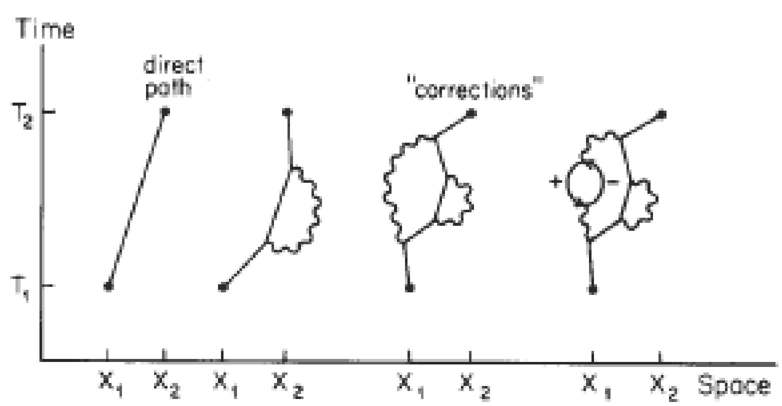
# From Path Integrals




# To Feynman Diagrams



Time Approach to Quantum Electrodynamics," "Mathematical Formulation of the Quantum Theory of Electromagnetic Interaction," and "An Operator Calculus Having Applications in Quantum Electrodynamics." As they appeared, the younger theorists who devoured them realized that Dyson had given only a bare summary of Feynman's vision. They felt invigorated by his images—beginning with the unforgettable bombardier metaphor in the positron paper—and by his way of insisting on the plainest statements of physical principles in physical language:

*The rest mass particles have is simply the work done in separating them against their mutual attraction after they are created. . . .*

*How would such a path appear to someone whose future gradually became past through a moving present? He would first see . . .*

No aspiring physicist could read these papers without thinking about what space was, what time was, what energy was. Feynman was helping physics live up to the special promise it made to its devotees: that this most fundamental of disciplines would bring them face to face with the primeval questions. Above all, however, to young physicists the diagrams spoke loudest.

Feynman had told Dyson, with a slight edge, that he had not bothered to read his papers. "Feynman and I really understand each other," Dyson wrote home cheerily. "I know that he is the one person in the world who has nothing to learn from what I have written; and he doesn't mind telling me so." Feynman's students, however, sometimes noticed what seemed to them an undercurrent of anger in the pointed comments he would make about Dyson. He had started hearing about Dyson's graphs—irritating. Why *graphs*? he asked Dyson. Was that the mathematician in him, putting on airs?

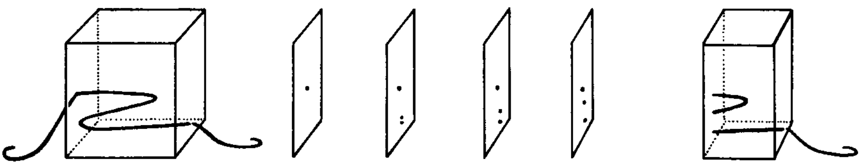
Feynman's space-time method had other antecedents besides Dyson's graphs, as it happened. A 1943 German textbook by Gregor Wentzel contained a parallel depiction of a particle exchange process in beta decay. A Swiss student of Wentzel's, Ernst Stückelberg, had developed a diagrammatic approach that even embraced the conception of time-reversed positrons; parts of this he published, in French, and parts were returned as unpublishable. (Wentzel himself was the unimpressed referee.) Their diagrams showed glimmerings of the style of visualization that Feynman now brought to fruition. His own full-dress version finally appeared in a paper he sent off in late spring 1949. "The fundamental interaction"—an image that would burn itself into the brains of the next generation of small factor. Weisskopf decided it was inconceivable that these brilliant upstarts could both be wrong, independently, and delayed his manuscript. Months passed before Feynman called apologetically to say that Weisskopf's answer had been correct.

For Feynman's own developing theory, a breakthrough came when he confronted the ticklish area of antimatter. The first antiparticle, the negative electron, or positron, had been born less than two decades earlier as a minus sign in Dirac's equations—a consequence of a symmetry between positive and negative energy. Dirac had been forced to conceive of holes in a sea of energy, noting in 1931 that "a hole, if there were one, would be a new kind of particle, unknown to experimental physics." Unknown for the next few months—then Carl Anderson, at Caltech, found the trail of one in a cloud chamber built to detect cosmic rays. It looked like an electron, but it swerved up through a magnetic field when it should have swerved down.

The vivid photographs, along with the lively name coined by a journal editor against Anderson's will, gave the positron a legitimacy that theorists found hard to ignore. The collision of an electron with its antimatter cousin released energy in the form of gamma rays. Alternatively, in Dirac's picture of the vacuum as a lively sea populated by occasional holes, or bubbles, one could say that the electron fell into a hole and filled it, so that both the hole and the electron would disappear. As experimentalists continued to study their cosmic-ray photographs, they also found the reverse process: a gamma ray, nothing more than a high-frequency particle of light, could spontaneously produce a pair of particles, one electron and one positron.

Dirac's picture had difficulties. As elsewhere in his physics, unwanted infinities arose. The simplest description of the vacuum, empty space at absolute zero, seemed to require infinite energy and infinite charge. And from the practical perspective of anyone trying to write proper equations, the infinitude of presumed particles caused infernal complications. Feynman, seeking a way out, turned again to the forward- and backward-flowing version of time in his work with Wheeler at Princeton. Once again he proposed a space-time picture in which the positron was a time-reversed electron. The geometry of this vision could hardly have been simpler, but it was so unfamiliar that Feynman strained for metaphors:

"Suppose a black thread be immersed in a cube of collodion, which is then hardened," he wrote. "Imagine the thread, although not necessarily quite straight, runs from top to bottom. The cube is now sliced horizontally into thin square layers, which are put together to form successive frames of a motion picture." Each slice, each cross-section, would show a dot,



and the dot would move about to reveal the path of the thread, instant by instant. Now suppose, he said, the thread doubled back on itself, "somewhat like the letter N." To the observer, seeing the successive slices but not the thread's entirety, the effect would resemble the production of a particle-antiparticle pair:

> In successive frames first there would be just one dot but suddenly two new ones would appear when the frames come from layers cutting the thread through the reversed section. They would all three move about for a while then two would come together and annihilate, leaving only a single dot in the final frames.

The usual equations of electron motion covered this model, he said, though it did require "a more tortuous path in space and time than one is used to considering." He remained dissatisfied with the analogy of the thread and kept looking for more intuitive ways to express his view, capturing as it did the essence of the distinction between seeing paths in time-bound slices and seeing them whole. A Cornell student who had served as a wartime bombardier had a suggestion, and the bombardier metaphor, the one Feynman finally published, became famous.

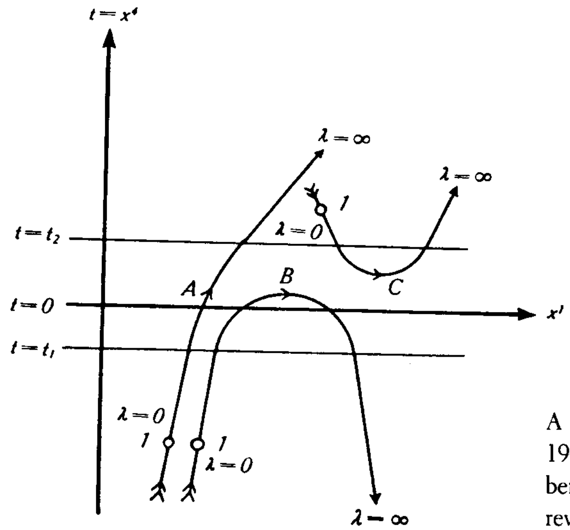
> A bombardier watching a single road through the bomb-sight of a low flying plane suddenly sees three roads, the confusion only resolving itself when two of them move together and disappear and he realizes he has only passed over a long reverse switchback of a single road. The reversed section represents the positron in analogy, which is first created along with an electron and then moves about and annihilates another electron.

That was the broad picture. His path-integral method suited the model well: he knew from his old work with Wheeler that the summing of the phases of nearby paths would applied to "negative time" as well. He also found a shortcut past complications that had arisen because of the Pauli exclusion principle, the essential law of quantum mechanics that forbade two electrons from inhabiting the same quantum state. He granted himself a bizarre dispensation from the exclusion principle on the basis that, where earlier calculations had seen two particles, there was actually just one, taking a zigzag back and forth through a slice of time. "Usual theory says no, because then at time between $t_y$, $t_x$ can't have 2 electrons in same state," he jotted in a note to himself. "We say it is same electron so Pauli exclusion doesn't operate." It sounded like something from the science fiction of time travel—hardly a notion designed for ready acceptance. He knew well that he was proposing a radical departure from the commonsense experience of time. He was violating the everyday intuition that the future does not yet exist and that the past has passed. All he could say was that time in physics had already departed from time in psychology—that nothing in the microscopic laws of physics seemed to mandate a distinction between past and future, and that Einstein had already ruined the notion of absolute time, independent of the observer. Yet Einstein had not imagined a particle's history reversing course and swerving back against the current. Feynman could only resort to an argument from utility: "It may prove useful in physics," he wrote, "to consider events in all of time at once and to imagine that we at each instant are only aware of those that lie behind us."

## *My Machines Came from Too Far Away*

Schwinger and Feynman were both looking ahead to the inevitable sequel to the elite Shelter Island meeting. A new gathering was planned for late March at a resort in the Pocono Mountains of Pennsylvania: again the setting was to be pastoral, the roster intimate, the agenda profound. Success had enhanced the already high-status guest list. Fermi, Bethe, Rabi, Teller, Wheeler, and von Neumann were returning, along with Oppenheimer as chairman, and now they would be joined by two giants of prewar physics, Dirac and Bohr.

They gathered on March 30, 1948, in a lounge under a tarnished green clock tower with a view over a golf course and fifty miles of rolling woodlands. The presentations opened with the latest news of particle tracks in cosmic-ray showers and in the accelerator at Berkeley. With its sixteen-foot

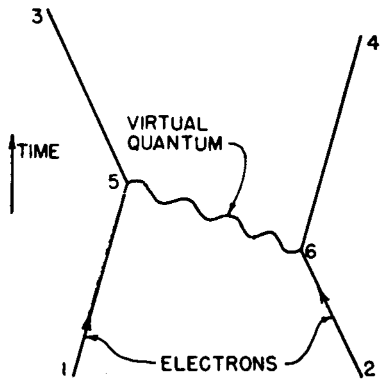



A diagram from a little-known 1941 paper of Ernst Stückelberg, showing a version of time reversal in particle trajectories.

field theorists—showed two electrons interacting by exchanging a single photon.

He drew electrons as solid lines with arrows. For photons he used wavy lines without arrows: no directionality needed because the photon's antiparticle is itself. "The fundamental interaction" reinterpreted the basic textbook process of electromagnetic repulsion. Two negative charges, electrons, repel. A standard picture, showing lines of force or merely two balls pressing apart from each other, would beg the question of how an entity feels the force of another entity at a distance. It would imply that force can be transmitted instantly, when in truth, as Feynman's diagrams automatically made explicit, whatever carries force can move only as fast as light. In the case of electromagnetism, it is light—in the form of fugitive "virtual" particles that flash into existence just long enough to help quantum theorists balance their books.

These were space-time diagrams, of course, representing time as one direction on the page. Typically the past sat at the bottom and the future at the top; one way to read the diagram would be to cover it with a sheet of paper, pull the paper slowly upward, and watch the history unfold. An electron changes course as it emits a photon. Another electron changes course when it absorbs the photon. Yet even the idea that the earlier event is *emission* and that the later is *absorption* represented a prejudice about time. It was built into the language. Feynman stressed how free his approach was from customary intuitions: these events are interchangeable.



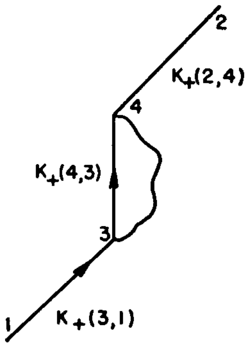

The Feynman diagram: "The fundamental interaction." It is a space-time diagram: the progress of time is shown upward on the page. If one covers it with a sheet of paper and then draws the paper slowly upward:

- A pair of electrons—their paths shown as solid lines—move toward each other.
- When (6) is reached, a virtual photon is emitted by the right-hand electron (wiggly line), and the electron is deflected outward.
- At (5) the photon is reabsorbed by the other electron, and it, too, is deflected outward.

Thus this diagram depicts the ordinary force of repulsion between two electrons as a force carried by a quantum of light. Because it is a *virtual* particle, coming into existence for a mere ghostly instant, it can temporarily violate the laws that govern the system as a whole—the exclusion principle or the conservation of energy, for example. And Feynman noted that it is arbitrary to think of the photon as being emitted in one place and absorbed in the other: one can say just as correctly that it is emitted at (5), travels *backward* in time, and is then (earlier) absorbed at (6).

The diagram is an aid to visualization. But it serves physicists mainly as a bookkeeping device. Each diagram is associated with a complex number, an *amplitude* that is squared to produce a probability for the process shown.

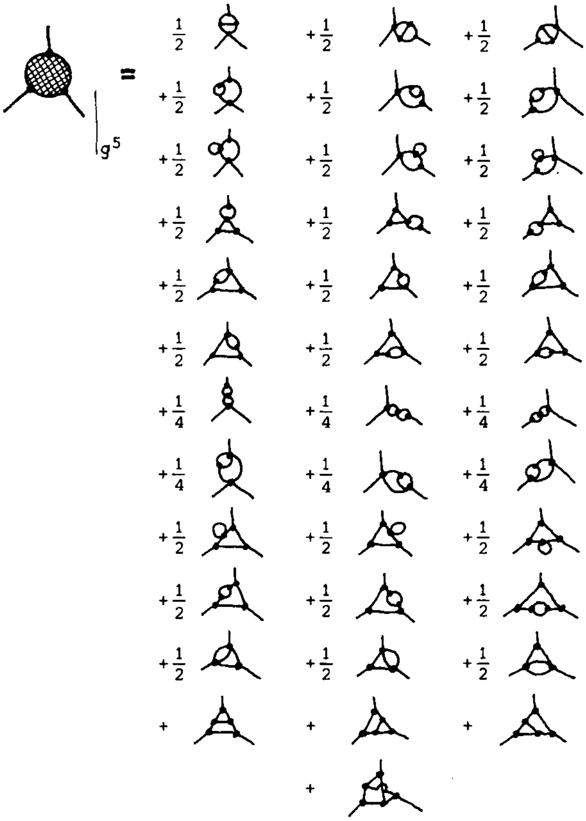
In fact each diagram represented not a particular path, with specified times and places, but a sum of all such paths. There were other simple diagrams. He represented the self-energy of an electron—its interaction with itself—by showing a photon line returning to the same electron that spawned it. There was a grammar of permissible diagrams, corresponding, as Dyson had emphasized, to the permissible mathematical operations. Still, the diagrams could grow arbitrarily complicated, virtual particles




Self-interaction. It is necessary to sum the amplitudes corresponding to many Feynman diagrams—to add the contributions for every way an event can occur. The continual possibility of virtual particles materializing and vanishing causes increasing complexity. Here an electron interacts with itself, in effect—the self-energy problem that first troubled Feynman in his work with Wheeler. It emits and absorbs its own virtual photon.

appearing and disappearing in an intricate, recursive mesh. Feynman's first H-shaped diagram for interacting electrons was the only such diagram with one virtual photon. Drawing all the possible diagrams with two virtual photons showed how quickly the permutations grew. Each made a contribution to the final computation, and more complicated diagrams became enormously difficult to calculate. Fortunately the greater the complication the less the probability and the smaller, therefore, the effect on the answer. Even so, physicists would shortly find themselves agonizing over pages of diagrams resembling catalogs of knots. They found it was worth the effort; each diagram could replace an effective lifetime of Schwingerian algebra.

Feynman diagrams seemed to depict particles, and they had sprung from a mind focused on a particle-centered style of visualization, but the theory they anchored—quantum field theory—gave center stage to the field. In a sense the paths of the diagrams, and the paths summed in the path integrals, were the paths of the field itself. Feynman read the *Physical Review* more avidly than ever in the past, watching for citations. For a while it was all Schwinger—a paper would be pages of glyphs and would culminate in a neat expression that Feynman felt he could simply have written down as a starting point. He was sure this could not last. It did not. Feynman's method, Feynman's rules, began to take over. In the summer of 1950 a paper appeared with small "Feynman diagrams" on the first page—"following the simplified methods introduced by Feynman." A month later came another: "a technique due to Feynman. . . . The calculation of matrix elements can be simplified greatly by use of the Feynman-Dyson methods." The unreasonable power of the diagrams in the hands of students frustrated some of the elders, who felt that physicists were waving a sword that they did not understand. As the flood of papers began to cite Feynman,



Making the increasingly precise calculations for which quantum electrodynamics became famous required formidable exercises in combinatorics.

Schwinger went into what he described as his retreat. "Like the silicon chip of more recent years, the Feynman diagram was bringing computation to the masses," he said. Later, people who overlooked the note of hoi polloi quoted this remark as though Schwinger had intended a tribute. He had not. This was "pedagogy, not physics."

> Yes, one can analyze experience into individual pieces of topology. But eventually one has to put it all together again. And then the piecemeal approach loses some of its attraction.

Schwinger's students at Harvard were put at a competitive disadvantage, or so it seemed to their fellows elsewhere, who suspected them of surreptitiously using the diagrams anyway. This was sometimes true. (They revered him, though—his night-owl ways, his Cadillac, his theatrically impeccable lecture performances. They emulated his way of saying, "We can effectively regard . . ." and they tried to construct the perfect Schwinger sentence: one graduate student, Jeremy Bernstein, liked a prototype that began, "Although 'one' is not perfectly 'zero,' we can effectively regard . . ." They also worried about Schwinger's ability to materialize silently beside them at the lunch table; a group of his graduate students protected themselves with a conversational convention in which *Schwinger* meant *Feynman* and *Feynman* meant *Schwinger*.)

Murray Gell-Mann later spent a semester staying in Schwinger's house in Cambridge and loved to say afterward that he had searched everywhere for the Feynman diagrams. He had not found any, but one room had been locked . . .

## *Away to a Fabulous Land*

Bethe worried that Feynman was growing restless after four years at Cornell. There were entanglements with women: Feynman pursued them and dropped them, or tried to, with increasingly public frustration—so it seemed even to undergraduates, who knew him as the least professorial of professors, likely to be found beating a rhythm on a dormitory bench or lying supine and greasy beneath his Oldsmobile. He had never settled into any house or apartment. One year he lived as faculty guest in a student residence. Often he would stay nights or weeks with married friends until these arrangements became sexually volatile. He seemed to think that Cornell was alternately too large and too small—an isolated village with only a diffuse interest in science outside the confines of its physics department. Furthermore, Hans Bethe would always be the great man of physics at Cornell.

An old Los Alamos acquaintance, Robert Bacher, after serving on the new Atomic Energy Commission, was moving to Caltech, where he was charged with rebuilding an obsolete-looking physics program. He was swimming in a lake during a summer vacation in northern Michigan when Feynman's name came into his head. He rushed back to shore, tracked Feynman down by telephone, and within a few days had him there visiting.

Feynman agreed to consider Pasadena, but he was also thinking about possibilities even more faraway, exotic, and warm. South America was on free electron; hence, they may be represented as $u \exp(-ip \cdot x)$, where $p$ is the energy ($p_4$) and momentum ($\mathbf{p}$) of the electron ($p^2 = m^2$), and $u$ is a constant 4-index symbol (a spinor, in modern terminology). If the wave functions are normalized to unit volume, the matrix element of the amplitude (14.6) between such states gives the first-order correction, $i(\Delta E)(t_2 - t_1)$, to the factor $\exp[-i(\Delta E)(t_2 - t_1)]$ in the amplitude for arrival in state $f(2)$ at time instant $t_2$, starting from the state $f(1)$ at time instant $t_1$. Thus the whole effect is equivalent to a change of the energy $\Delta E$, given by the expression

$$\Delta E = e^2 \int (\bar{u}\gamma_\mu K_+(4, 3)\gamma_\mu u) \exp(ip \cdot x_{43})\, \delta_+(s_{43}^2)\, d\tau_4. \qquad (14.7)$$

Similarly, one can obtain an expression for the energy shift in the hydrogen atom. For this purpose, one needs to replace the amplitude for the free electron $K_+$ in formula (14.7) with the corresponding amplitude $K_+$ of the electron in the potential $V = \beta e^2/r$ of the atom, and the free state $f$ by a wave function (of space and time) for an atomic state. The real evaluation of expressions like (14.7) can be performed more easily by the technique which Feynman described next.

## 14.4 Expression in momentum and energy space: the Feynman diagrams

The calculation of matrix elements is most simple in the energy and momentum representation. The reason is quite simple. In this representation all Hamiltonian equations for free particles, like the Dirac equation, the Klein–Gordon equation, and Maxwell's equations, transform into algebraic equations which have very simple solutions. Taking into account the formula (13.32) for the Fourier transform of the $K_+$ function, the formula (13.33) for the Fourier transform of the electromagnetic four-potential, and the formula for the Fourier transform of the $\delta_+$-function,

$$-\delta_+(s_{21}^2) = \pi^{-1} \int \exp(-ik \cdot x_{21}) k^{-2}\, d^4k. \qquad (14.8)$$

we can rewrite all the expressions for the quantum amplitude, or its matrix elements, in the momentum and energy representation. The resulting formulas need to be justfied by some rules for giving unambiguous meaning to the corresponding expressions. For example, in equation (14.8) we have to understand the term $k^{-2}$ as the limit $\varepsilon \to +0$ of $(k_\mu k_\mu + i\varepsilon)^{-1}$,[11] i.e. certain rules for going around the poles of the corresponding singular functions in the Fourier integrals are needed. Already in paper I,[2] Feynman had proved that such rules are equivalent to the proper choice of the solution of the quantum dynamical equations. For example, in equation (11.32), one can understand the function $(p-m)^{-1}$ as $(p-m+i\varepsilon)^{-1}$, with $\varepsilon \to +0$, to obtain just the Feynman propagator $K_+$.[12] All such rules can be expressed in a general rule, according to which the masses of all particles and quanta have infinitesimal negative imaginary parts.

Using these rules, Feynman represented the self-energy, equation (14.8), as a matrix between $\bar{u}$ and $u$ of the matrix

$$(e^2/\pi i)\int \gamma_\mu (p-k-m)^{-1}\gamma_\mu k^{-2}\, d^4k, \tag{14.9}$$

which obviously has quite a simple form, and wrote: 'The equation [(14.9)] can be understood by imagining [see Fig. 14.3] that the electron of momentum $p$ emits $(\gamma_\mu)$ a quantum of momentum $k$, and makes its way now with momentum $p-k$ to the next event [factor $(p-k-m)^{-1}$] which is to absorb the quantum [another $(\gamma_\mu)$]. The amplitude of propagation of quanta is $k^{-2}$. (There is a factor $e^2/\pi i$ for each virtual quantum.) One integrates over all quanta. The reason an electron of momentum $p$ propagates as $1/(p-m)$ is that this operator is reciprocal to the Dirac equation operator, and we are simply solving this equation. Likewise light goes as $1/k^2$, for this is the reciprocal Dalembertian operator of the wave equation of light. The first $\gamma_\mu$ represents the current which generates the vector potential, while the second is the velocity operator by which this potential is multiplied in the Dirac equation when an external field acts on an electron.'[12]

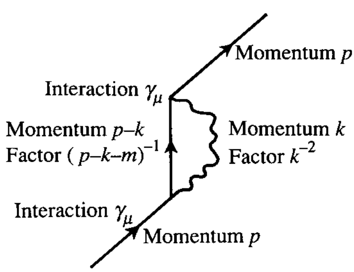



Fig. 14.3. Interaction of an electron with itself. Momentum space, equation (14.9).

These rules permit one to describe via the corresponding diagrams every process in quantum electrodynamics and to write down the matrix elements of the amplitude of this process. Today, these rules are the most well known part of Feynman's approach to quantum electrodynamics and, in general, to field theory and statistical physics. The practical usefulness of the Feynman rules and diagrams made them one of the most essential elements of the scientific training of every theoretical physicist.

As Feynman recalled many years later: '. . . The diagrams were intended to represent physical processes and the mathematical expressions used to describe them. Each diagram signified a mathematical expression. In these diagrams I was seeing things that happened in space and time. Mathematical quantities were associated with points in space and time. I would see electrons going along, being scattered at one point, then going over to another point and getting scattered there, emitting a photon and the photon goes over there. I would make little pictures of all that was going on; these were physical pictures involving the mathematical terms. These pictures evolved only gradually in my mind. There were some old pictures that were quite similar, but not as clean and as final as the diagrams I was drawing; they became a shorthand for the processes I was trying to describe physically and mathematically.

'The diagrams became very important as I began to treat more and more of these problems. They became pictorial representations of the more and more abstract things I was trying to describe. I remember that when I was at the Telluride House (at Cornell) and was working on the self-energy of the electron, there were many terms which I was trying to visualize, when it occurred to me that these pictures looked very funny. In ancient Egypt and Greece the priests and oracles used to look at the veins in sheep's livers to forecast the future, and that's the kind of pictures I was drawing to describe physical phenomena. I thought that if they really turn out to be useful it would be fun to see them in the pages of the *Physical Review*. I was conscious of the thought that it would be amusing to see these funny-looking pictures in the *Physical Review*.'[13]

The Feynman rules for the matrix elements in spinor electrodynamics in modern form are summarized in Table 14.1.[14] This is just Feynman's 'handbook on how to do quantum electrodynamics'.[15]

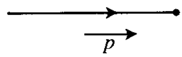
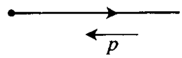
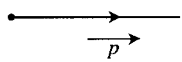
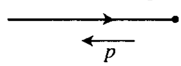
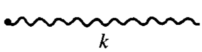
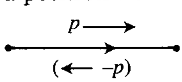
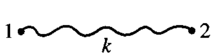
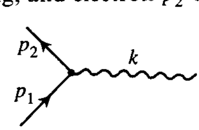
By making use of these rules, Feynman wrote down the matrix elements for a different kind of process in quantum electrodynamics. For example, the total matrix element for the Compton scattering of an electron in second-order perturbation theory is then

$$e_2(p_1+q_1-m)^{-1}e_1+e_1(p_1+q_2-m)^{-1}e_2. \tag{14.10}$$

According to Feynman's rules this expression corresponds to two possible diagrams for Compton scattering as shown in Fig. 14.4. One has to take the matrix elements of the expression (14.10) to obtain the Klein–Nishina formula.

For the radiative corrections to the scattering of the electron in the lowest order of perturbation theory, Feynman gave three diagrams, which are shown in Fig. 14.5.[16] The three diagrams differ in the ordering of the processes of scattering of the electron, and of the emission and absorption of the photon. Feynman remarked that the expressions so obtained for various processes 'are, as has been indicated, no more than the re-expression of conventional quantum electrodynamics. As a consequence, many of them are meaningless. For example, the self-energy expression [(14.7)] or [(14.9)] gives an infinite

Table 14.1. Feynman's Rules

| | Element of the Feynman's diagram | Factor in the matrix element |
|---|---|---|
| 1 | Electron in the initial state with momentum $p$   | $(2\pi^{-3/2}u^{s,-}(p)$ |
| 2 | Positron in the initial state with momentum $p$   | $(2\pi^{-3/2}\bar{u}^{s,-}(p)$ |
| 3 | Electron in the final state with momentum $p$   | $(2\pi^{-3/2}\bar{u}^{s,+}(p)$ |
| 4 | Positron in the final state with momentum $p$   | $(2\pi^{-3/2}u^{s,+}(p)$ |
| 5 | Photon in the initial or final state with polarization $e_\nu$ and momentum $k$   | $\dfrac{e_\mu^\nu}{(2\pi)^{3/2}\sqrt{2k_0}}\ (\nu \neq 0)$ |
| 6 | Motion of an electron from 1 to 2 (or of a positron from 2 to 1)   | $\dfrac{1}{(2\pi)^4 i}\displaystyle\int d^4p\,\dfrac{m+p}{m^2-p^2-i\varepsilon}$ |
| 7 | Motion of a photon between vertices with summation indices $\mu$ and $\nu$   | $\dfrac{q^{\mu\nu}}{(2\pi)^4 i}\displaystyle\int d^4k\,\dfrac{1}{k^2+i\varepsilon}$ |
| 8 | Vertex with summation index $\nu$ with electron line $p_1$ and photon line $k$ incoming, and electron $p_2$ outgoing   | i.e. $\gamma^\nu(2\pi)^4\delta(p_2-p_1-k)$ |

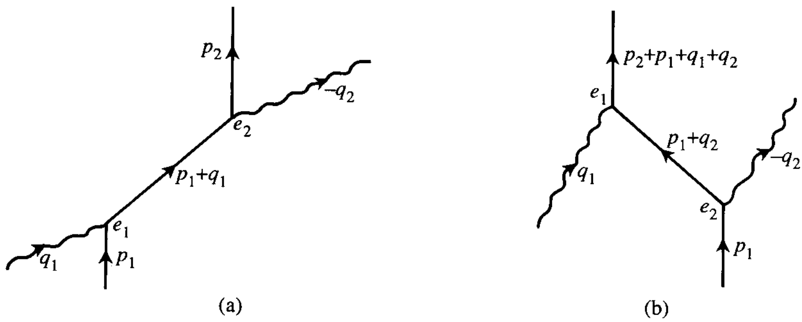



Fig. 14.4. Compton scattering, equation (14.10).

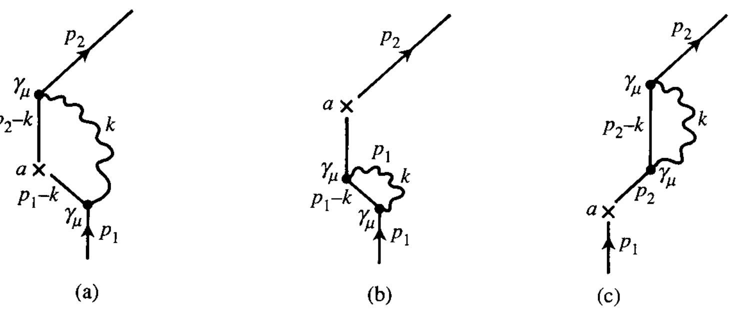



Fig. 14.5. Radiative correction to scattering, momentum space.

result when calculated. The infinity arises, apparently, from the coincidence of the δ-function singularities in $K_+(4,3)$ and $\delta(s_{43}^2)$. Only at this point is it necessary to make a real departure from conventional electrodynamics, a departure other than simply rewriting expressions in a simpler form.'[16]

In order to overcome the difficulties with the divergences connected with virtual quanta, Feynman made use of the regularization procedure which he had invented earlier[5, 17] (see Sections 13.1 and 13.2). Using this procedure with a little modification, he obtained for the expression (14.10), which gives the self-energy of the free electron, the result,

$$(e^2/2\pi)\{4m[\ln(\lambda/m)+\tfrac{1}{2}]-p[\ln(\lambda/m)+\tfrac{5}{4}]\}, \qquad (14.11)$$

up to terms in higher order of the ratio $\lambda/m$ of Feynman's cut-off parameter $\lambda$ and the mass $m$ of the electron. When applied to the state of the free electron with momentum $p$, satisfying the Dirac equation, the equation (14.7) gives the change of the mass,

$$\Delta m = m(e^2/2\pi)\,[3(\ln(\lambda/m)+\tfrac{3}{4}]. \qquad (14.12)$$

For the radiation corrections to the scattering, the sum of three terms (which corresponds to the three Feynman diagrams in Fig. 14.5 leads [for small momentum transfer $q$ ($\sqrt{q^2} = 2m \sin\theta$, $\theta$ being the scattering angle)] the result:

$$(e^2/2\pi)\left\{\frac{1}{2m}(qa - aq) + \frac{4q^2}{3m^2}a\left[\ln\left(\frac{m}{\lambda_{\min}}\right) - \frac{3}{8}\right]\right\}. \quad (14.13)$$

Here $a$ is the amplitude of the four-dimensional electromagnetic potential $A = A_\mu\gamma_\mu$, which is supposed to be of the form $a\exp(-iqx)$, and $\lambda_{\min}$ is the small mass of the proton: $\lambda_{\min} < m < \lambda$, which has to be introduced to avoid infrared divergences, according to Bloch and Nordsieck (see Section 11.4). This formula shows the change of the magnetic moment of the electron in accordance with Schwinger's result, and the Lamb shift as was interpreted in greater detail by Feynman previously[17] (see Section 13.2). It is remarkable that the result (14.13) does not depend on the regularization procedure at all, assuming that the mass $m$ is the experimental mass of the electron.

Feynman then discussed some of the difficulties of his regularization procedure, which is good enough only in the limit when $\lambda$ goes to infinity, after mass renormalization. He concluded: 'I have no proof of the mathematical consistency of this procedure, but the presumption is very strong that it is satisfactory.'[18]

## 14.5 The problem of vacuum polarization

In his article on the 'Space–time approach to quantum electrodynamics', Feynman published for the first time his point of view on the problem of vacuum polarization. The problem arises, for instance, in the analysis of the radiative corrections to scattering, where one type of term was not considered until now. The potential, on which the electron scatters, was assumed to vary as $a_\mu \exp(ip\cdot x)$, and the direct scattering on this external potential—in the lowest order of perturbation theory—corresponds to the diagram shown in Fig. 14.6(a). This diagram is a part of the diagrams shown in Fig. 14.5. But the same potential may create an electron–positron pair (see Fig. 14.6(b)) which then reannihilates, emitting a quantum with momentum $q = p_a - p_b$. This quantum scatters the original electron from state 1 to state 2.

The matrix element connected with the diagram for the process indicated in Fig. 14.6(b) is given by the expression

$$-(e^2/\pi i)\,\bar{u}_2\gamma_\mu u_1)\int S_p[(p_a + q - m)^{-1}\gamma_\nu(p_a - m)^{-1}\gamma_\mu]q^{-2}a_\nu\, d^4p, \quad (14.14)$$

where no regularization has been made. One can imagine that the closed loop of the electron–positron pair is equivalent to the current

NR PI → FEYNMAN DIAGRAMS

FIRST GENERALIZED MOMENTUM

$$\pi = \vec{p} - \frac{e}{c}\vec{A}$$

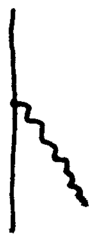
$\vec{p}$ ← PARTICLE MOMENTUM

$\frac{e}{c}\vec{A}$ ← FIELD MOMENTUM

$$KE = \frac{\vec{\pi}^2}{2m} = \frac{\vec{p}^2 + \frac{e}{c}\vec{p}\cdot\vec{A} + \frac{e}{c}\vec{A}\cdot\vec{p} + \frac{e^2}{c^2}\vec{A}^2}{2m}$$

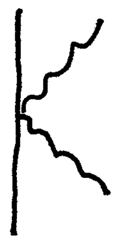
PHOTON EMISSION

$\vec{p}\cdot\vec{A}$



PHOTON ABSORPTION

$\vec{A}\cdot\vec{p}$



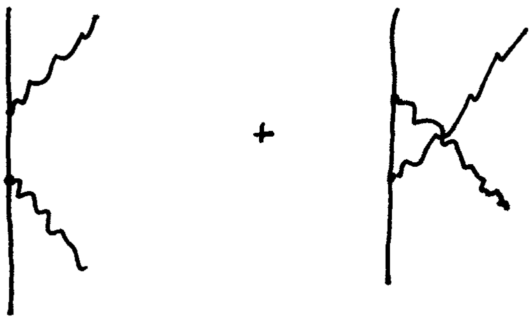
PHOTON SCATTERING    THOMSON SCATTERING

$A^2$

$\langle f \mid \vec{p} \cdot \vec{A} \mid i \rangle$ PROB AMP TO EMIT A PHOTON

SEAGULL DIAGRAM IS NOT COVARIANT

⟸ SAME TIME NO SIMULTANEITY



COMPTON SCATTERING

ALL POSSIBLE TIMES

ALL POSSIBLE PLACES

EVERY POINT IN SPACE TIME

⇒ MANIFESTLY COVARIANT

Thus we can get the correct answer for the probability of partial reflection by imagining (falsely) that all reflection comes only from the front and back surfaces. In this intuitively easy analysis, the front surface and back surface arrows are mathematical constructions that give us the right answer, whereas the analysis we just did---with the spacetime drawing and the arrows forming a part of a circle---is a more accurate representation of what is really going on: partial reflection is the scattering of light by electrons **inside** the glass.

Now what about the light that goes **thru** the layer of glass? First there is an amplitude that the photon goes straight thru the glass without hitting any electrons. This is the most important arrow in terms of length. But there are six other ways that a photon could reach the detector below the glass: a photon could hit X1 and scatter down to the detector, a photon could hit X2, X3, X4, X5, X6 ... These arrows have the same length as the arrows that formed the circle in the previous example

The same effect would appear if photons went slower thru glass than thru air: there would be extra turnings of the final arrow. That's why I said earlier that light appears to go slower thru glass (or water) than thru air. In reality the "slowing' of the light is extra turning caused by the atom in the glass (or water) scattering the light. The degree to which there is extra turning of the final arrow as the light goes thru a given material is called its index of refraction.

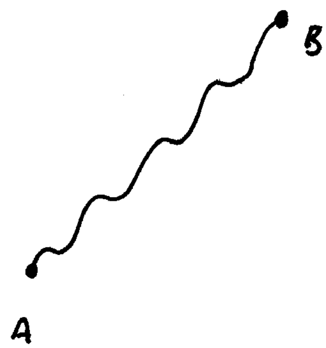
It is hard to believe that the vast apparent variety of Nature results from the monotony of repeatedly combining just these three basic actions. But it does.

Throughout these lectures I have delighted in showing you that the price of gaining such an accurate theory has been the erosion of our common sense. We must accept some very bizarre behavior: the amplification and suppression of probabilities, light reflecting from all parts of a mirror, light travelling in paths other than a straight line, photons going faster or slower than the conventional speed of light, electrons going backwards in time, photons suddenly disintrigrating into a positron-electron pair, and so on. That we must do, in order to appreciate what Nature is really doing underneath nearly all of the phenomena we see in the world.

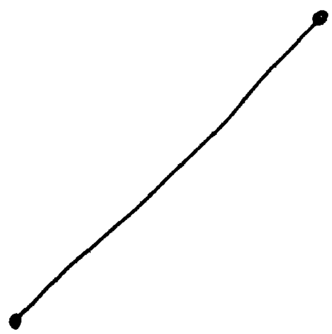
With the exception of technical details of polarization, I have described to you the framework by which we understand all of these phenomena. We draw **amplitudes** for every way an event can happen and add them when we would expect to add probabilities under normal circumstances; we multiply amplitudes when we would have expected to multiply probabilities. Thinking of everything in terms of amplitudes may cause difficulty at first because of their abstraction, but after a while, one gets used to this strange language. Underneath so many of the phenomena we see every day are only three basic actions: one is described by the simple coupling number j; the other two by functions---P(a to b) and E(a to b)---both of which are closely related. That's all there is to it, and from it all of the rest of the laws of physics come.

However before I finish this lecture, I would like to make a few additional remarks. One can understand the spirit of quantum electrodynamics without including this technical detail of polarization. But I'm sure that you will all feel uncomfortable unless I say something about what I've been leaving out. Photons it turns out come in four diferent varieties, called polarizations, that are related geometrically to the directions of space and time. Thus there are photons polarized in the X, Y, Z, and T directions. Perhaps you have heard that light comes in only two states of polarization---for example, a photon going in the Z direction can be polarized at right angles, either in the X or the Y direction. Well, you guessed it: in situations where the photon goes a long distance and appears to go at the speed of light, the amplitudes for the Z and the T terms exactly cancel out. But for virtual photons going between a proton and an

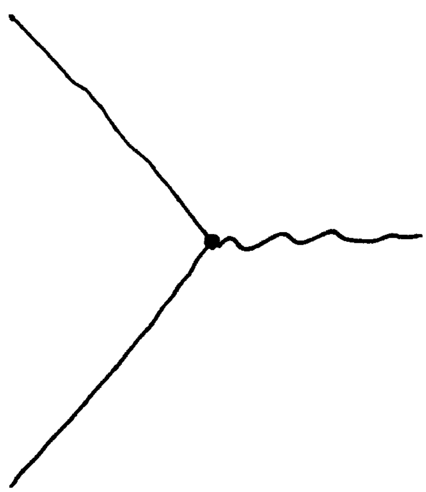
THREE ACTIONS FOR QED "A PARTICLE PICTURE"




PHOTON P(A → B)



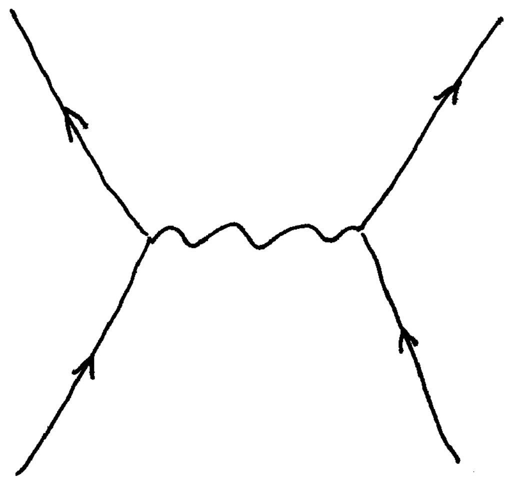
ELECTRON E(A → B)



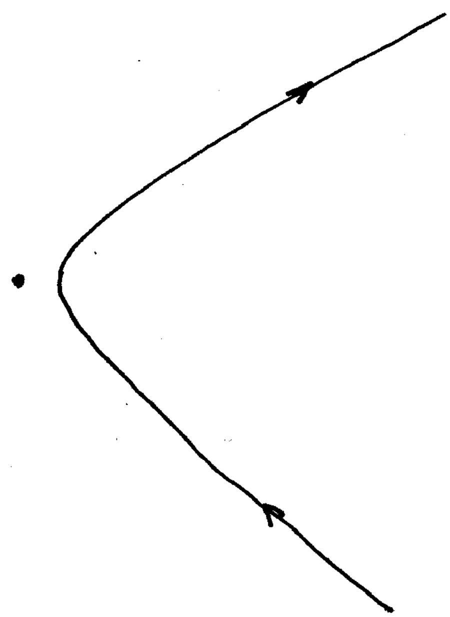
ELECTRON EMITS OR ABSORBS A PHOTON

$j \approx -0.1$

NO ELECTRIC FIELDS
NO MAGNETIC FIELDS



VIRTUAL PHOTON EXCHANGE

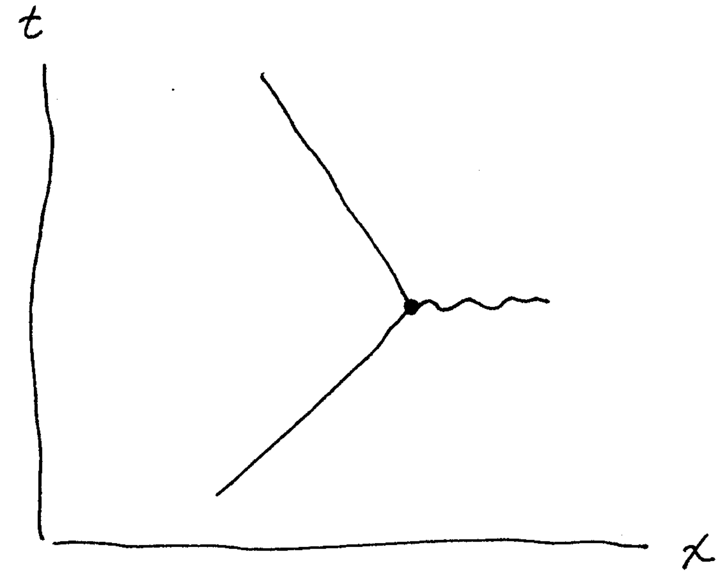


COULOMB POTENTIAL

$\Delta E \, \Delta t \sim \hbar/2$

BREMSSTRALUNG

ACTION 3: AN ELECTRON EMITS OR ABSORBS A PHOTON




COUPLING $j \approx -0.1$

SAME TO EMIT OR ABSORB

FOR THE PHOTON

$$ds^2 = (c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = 0$$

OVER LARGE DISTANCES $v = c$ REAL PHOTONS

BUT OVER SHORT DISTANCES

$v > c$ and $v < c$ } BOTH CONTRIBUTE VIRTUAL PHOTONS

over large distances these average to zero.

FOR THE ELECTRON $E(a \to b)$

$E(a \to b)$ IS SIMILAR TO $P(a \to b)$

$$E(a \to b) = \underbrace{P(a \to b)}_{\text{NO HOPS}} + \underbrace{P(a \to c)\, m^2\, P(c \to b)}_{\text{1 HOP}}$$

$$+ \underbrace{P(a \to d)\, m^2\, P(d \to e)\, m^2\, P(e \to b)}_{\text{2 HOPS}}$$

$$+ \; 3, 4, 5, \ldots \text{ HOPS}$$

if $m = 0$ $\quad E(a \to b) = P(a \to b)$

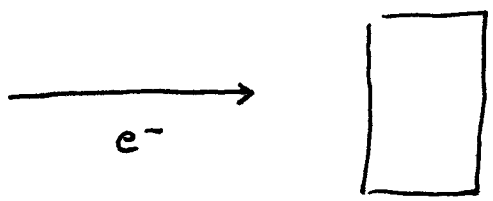
AT LARGE DISTANCES, ONLY REAL ELECTRONS

$$E^2 = p^2 c^2 + m_e c^4$$

AT SMALL DISTANCES, VIRTUAL ELECTRONS

$$E^2 \neq p^2 c^2 + m_e c^4$$

# BREMSSTRAHLUNG

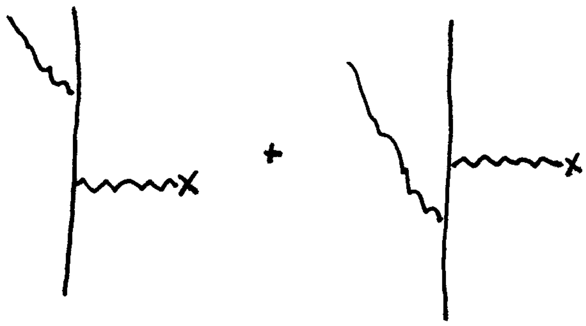



SUDDEN STOP

LARGE ACCELERATION

LOTS OF RADIATION

QED




SAME FINAL STATE

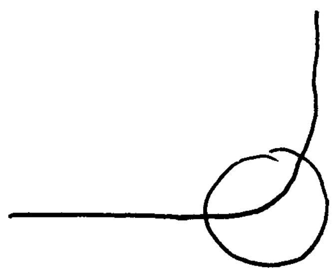
NUCLEUS SEATHING WITH VIRTUAL ~~ELECTRONS~~ PHOTONS

COLLISION WITH ELECTRON MAKES ONE REAL

ONLY REAL PHOTONS TRAVEL TO $\infty$

PAIR PRODUCTION NEAR NUCLEI

SYNCHROTRON RADIATION

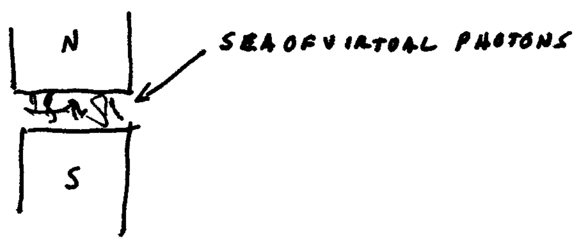


SHARP BEND

LARGE ACCELERATION

LOTS OF RADIATION

QED




COLLISION WITH e MAKES ONE REAL

PHOTON $E = pc$ $E^2 = p^2c^2$ REAL

$E \neq pc$ VIRTUAL

ELECTRONS $E^2 = p^2c^2 + m_e^2c^4$ REAL

$E^2 \neq$ VIRTUAL

OFF MASS SHELL

REAL PHOTONS LIVE FOREVER — MACROSCOPIC DISTANCE

" ELECTRONS " "

VIRTUAL PARTICLES DO NOT

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

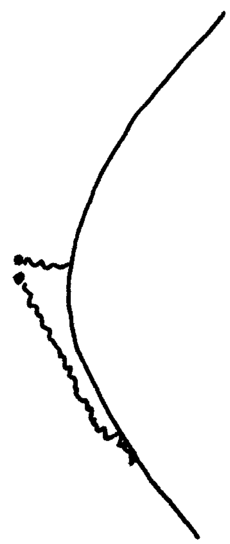
$$\Delta t \approx \frac{\hbar}{2 \Delta E}$$

$$d = v t = c \frac{\hbar}{2E}$$

LARGE ENERGY => SHORT DISTANCE

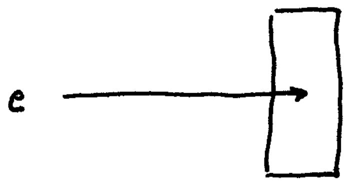
SMALL ENERGY => LONG DISTANCE

COULOMB SCATTERING

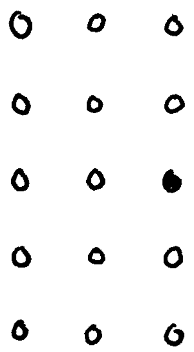


FORCE BETWEEN 2 ELECTRONS ...

# BREMSSTRAHLUNG HOLOGRAPHY




CRYSTAL

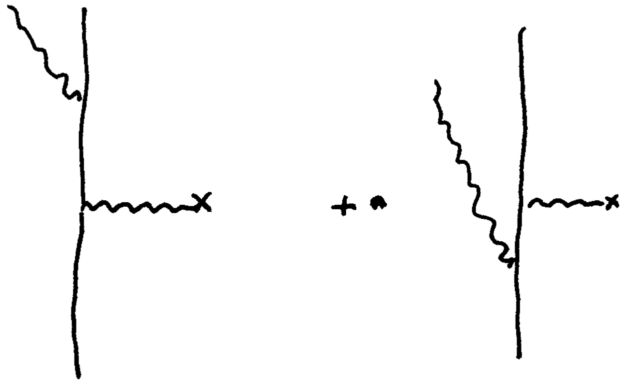


INTERFERENCE PATTERN

$\Downarrow$

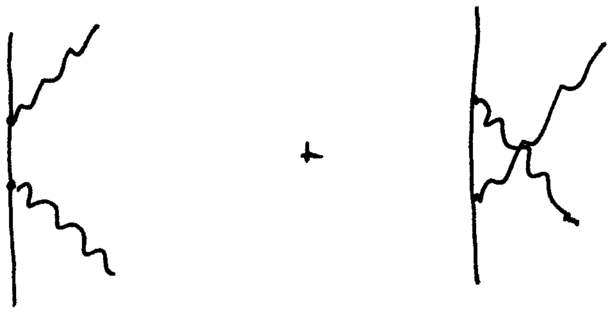
CRYSTAL STRUCTURE

BREMSSTRAHLUNG SOURCE



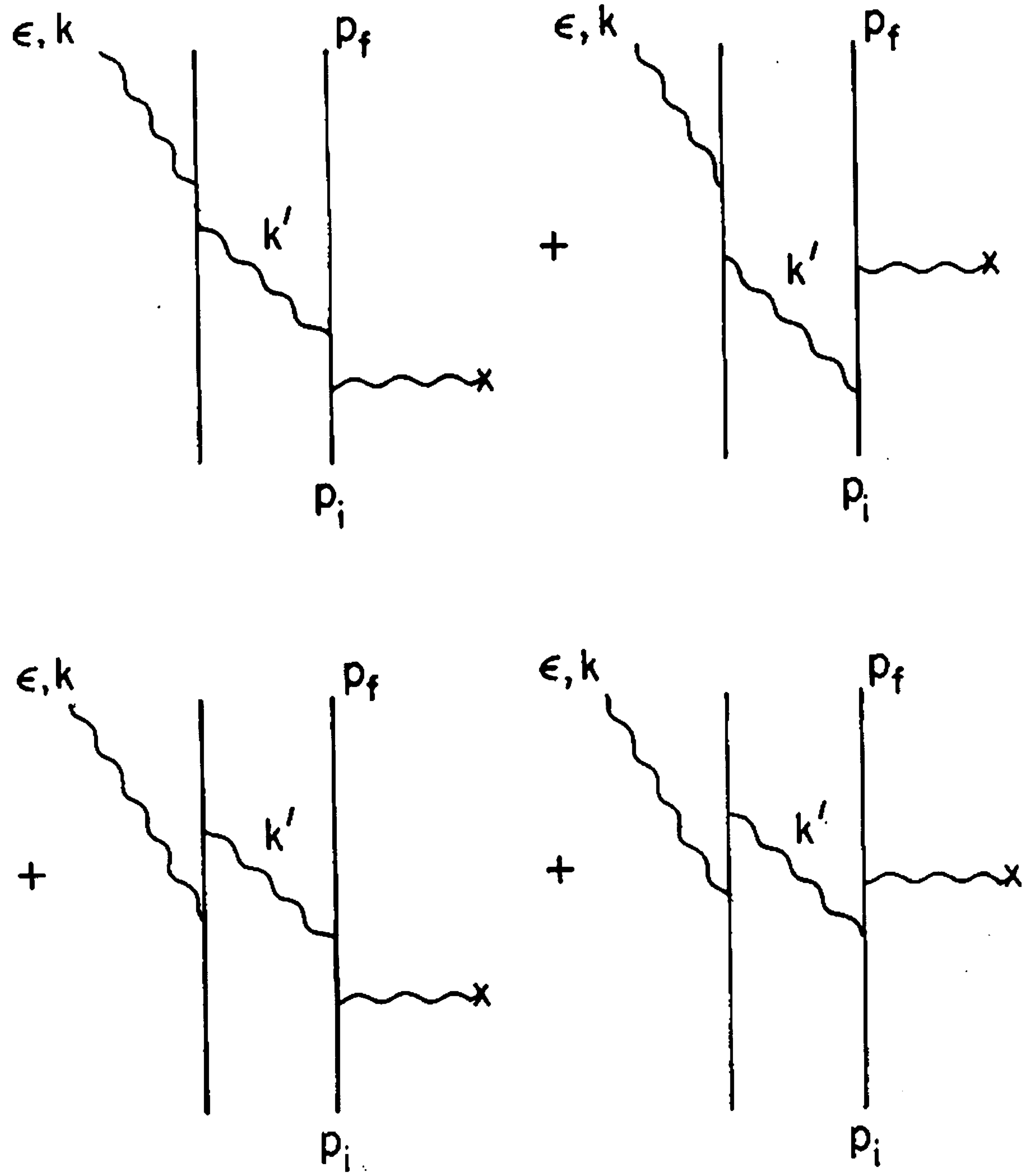
SAME FINAL STATE $\Rightarrow$ INTERFERE

COMPTON SCATTERING



SAME FINAL STATE $\Rightarrow$ INTERFERE

FOUR DIAGRAMS



ε, k
p_f
k′
x
p_i
+
ε, k
p_f
k′
x
p_i
ε, k
p_f
+
k′
x
p_i
ε, k
p_f
+
k′
x
p_i

$$\mathcal{O} = -Z^2 e^4 e_p \int \langle C^\mu(\epsilon,k,k')\rangle \frac{\widetilde{B}_\mu(k')}{\omega^2 - \vec{k}'^2 + i\epsilon} \times e^{-i(\vec{k}-\vec{k}')\cdot\vec{r}} \frac{d^3k'}{(2\pi)^3} \frac{[1 - F(|\vec{q}'|)]}{\vec{q}'^2 + i\epsilon},$$

where $\vec{q}' = \vec{p}_f - \vec{p}_i + \vec{k}'$. Note that $k'^2 = \omega^2 - \vec{k}'^2 \neq 0$.

JUST LIKE DIRAC NOTATION, AT SOME POINT SOMEONE MUST DO THE INTEGRALS...

PROPAGATORS


$$\int d^4 k \quad \frac{1}{k^2 + i\epsilon}$$


$$\int d^4 p \quad \frac{m + p}{m^2 - p^2 - i\epsilon}$$

PRB 56, 2399 (1997).

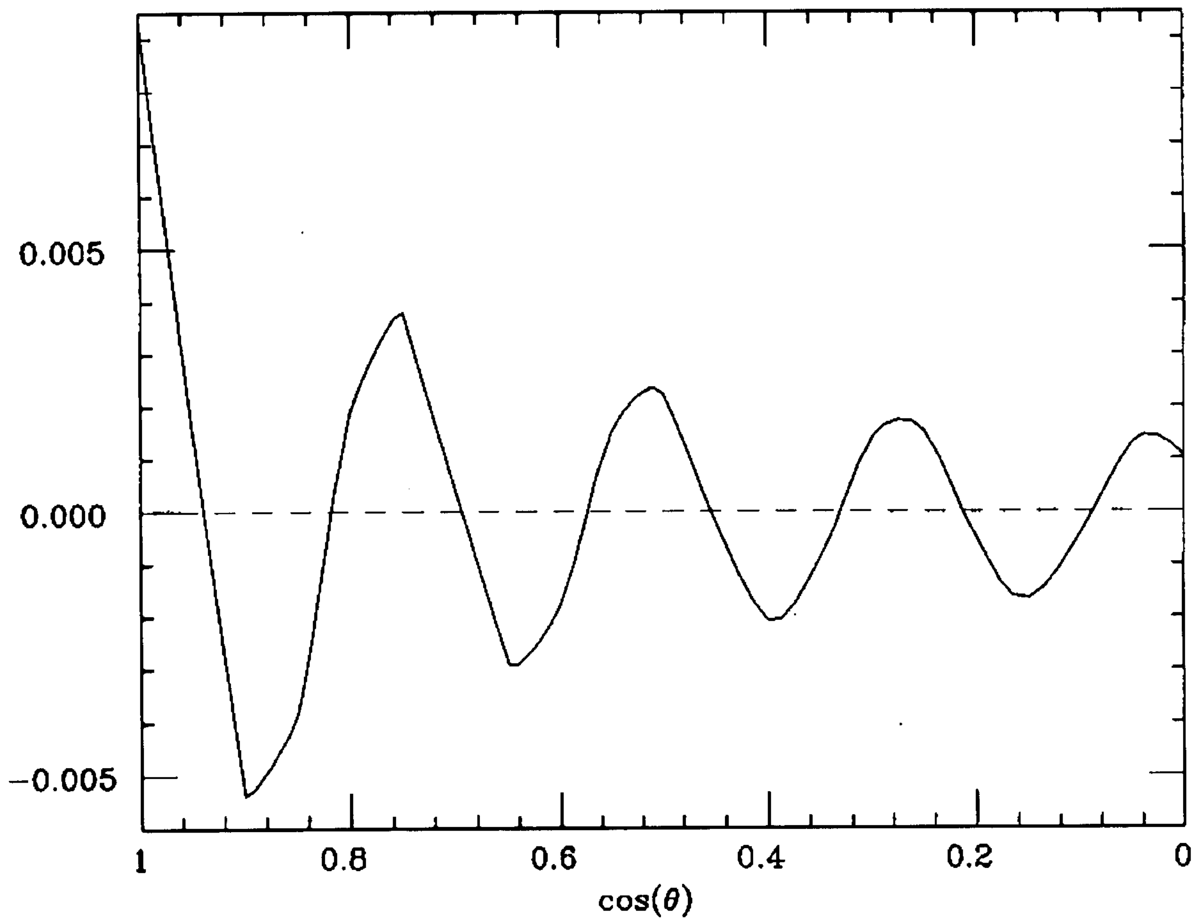



FIG. 8. The small effects of virtual electron propagation in the uncrossed graph in Fig. 2. The real part of the uncrossed correction $\delta J_{\rm on}^{\rm uncr}$ (dashed line) given by Eq. (57) is compared with the real part of the on-shell separated atom approximation $J_{\rm on}$ (solid line) given by Eq. (27). Here $\hat{k}\cdot\hat{p}_f=0.5$ and $\hat{p}_i\cdot\hat{r}=0.5$.

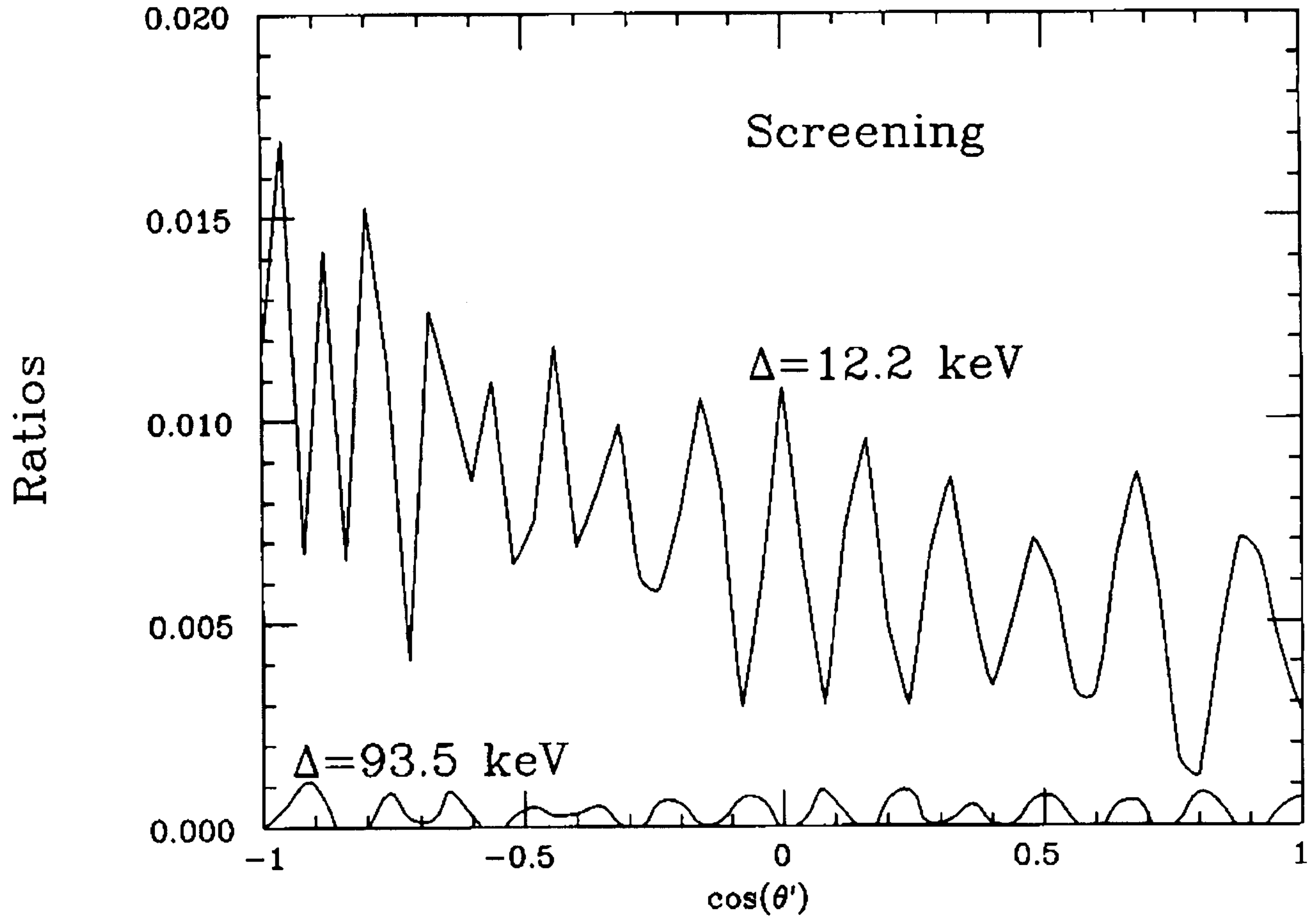



FIG. 6. The small effects of photon virtuality on the screening correction given by the ratio of the second to the first term in Eq. (40). The ratio $\mathrm{Re}\,\delta I_s/\mathrm{Re}\{[1-F(q_2)]J_{\mathrm{on}}\}$ is plotted to illustrate the size of these corrections for two typical experimental values of the momentum transfer, namely 12.2 keV and 93.5 keV. Here the momentum transfer $\vec{\Delta}\equiv\vec{p}_i-\vec{p}_f$ and the angle $\theta'$ is specified by $\cos(\theta')=\hat{\Delta}\cdot\hat{r}$.